Kittel sstate

6.1 (a) $U_0 = \frac{3}{5} N \varepsilon_F$, $\quad p = -\frac{\partial U_0}{\partial V} = -\frac{3}{5} N \frac{\partial \varepsilon_F}{\partial V}$.

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3N\pi^2}{V}\right)^{2/3}$$

$$= \frac{\hbar^2}{2m} k_F^2 \quad \Rightarrow \quad \frac{\partial \varepsilon_F}{\partial V} = \frac{\hbar^2}{m} k_F \frac{\partial k_F}{\partial V}.$$

$$k_F = \left(\frac{3N\pi^2}{V}\right)^{1/3} = \left(3N\pi^2\right)^{1/3} V^{-1/3}.$$

$$\Rightarrow \frac{\partial k_F}{\partial V} = \left(3N\pi^2\right)^{1/3}\left(-\frac{1}{3}\right) V^{-4/3}.$$

$$= \left(3N\pi^2\right)^{1/3}\left(-\frac{1}{3}\right) V^{-1/3} \frac{1}{V}$$

$$= -\frac{1}{3} k_F \frac{1}{V}.$$

$$\Rightarrow \frac{\partial \varepsilon_F}{\partial V} = \frac{\hbar^2}{m} k_F \left(-\frac{1}{3} k_F \frac{1}{V}\right) = -\frac{1}{3V} \frac{\hbar^2 k_F^2}{m}$$

$$= -\frac{2}{3V} \varepsilon_F.$$

$$\Rightarrow p = -\frac{3}{5} N \frac{\partial \varepsilon_F}{\partial V} = \frac{3}{5} N \frac{2}{3} \varepsilon_F \frac{1}{V}.$$

$$= \boxed{\frac{2}{3V} U_0}$$

Davidson Cheng
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Q. 2 (b) $B = -V \frac{\partial P}{\partial V} = -V \frac{\partial}{\partial V}\left[\frac{2}{3V} U_0\right]$

$$= -\frac{2}{3} V \frac{\partial}{\partial V}\left[U_0 / V \right].$$

$$= -\frac{2}{3} V \left[\frac{\partial U_0}{\partial V}\frac{1}{V} - \frac{U_0}{V^2} \right].$$

Recall $\frac{\partial U_0}{\partial V} = -p = -\frac{2}{3}\frac{U_0}{V}.$

$$\Rightarrow \frac{\partial U_0}{\partial V}\frac{1}{V} - \frac{U_0}{V^2} = -\frac{2}{3}\frac{U_0}{V^2} - \frac{U_0}{V^2}$$

$$= -\frac{5}{3}\frac{U_0}{V^2}$$

$$\Rightarrow -\frac{2}{3} V \left[\frac{\partial U_0}{\partial V}\frac{1}{V} - \frac{U_0}{V^2} \right] = -\frac{2}{3} V \left[-\frac{5}{3}\frac{U_0}{V^2} \right]$$

$$= \boxed{\frac{10}{9}\frac{U_0}{V}} = B.$$

(c) We found. $B = \frac{10}{9}\frac{U_0}{V}$, using $U_0 = \frac{3}{5} N \varepsilon_F$, we have

$$B = \frac{10}{9}\frac{3}{5}\frac{N}{V}\varepsilon_F = \frac{2}{3} n \varepsilon_F \quad \text{where } n = \frac{N}{V}.$$

For potassium (K), $\varepsilon_F = 2.12\ eV$, $n = 1.4\ cm^{-3}$.

$$\Rightarrow \boxed{B = 1.96\ \frac{eV}{cm^3}}.$$

Davidson Chang
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